Topological characteristics of economic transaction networks

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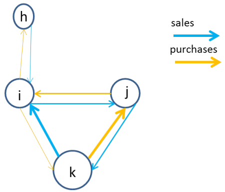
Outline

Introduction

- Legislation on Value Added Tax (VAT) requires buyers and sellers to communicate their exchanges.
- This information can be used to study the network structures underlying these trade relations.

- Both statements should coincide. If not, an irregularity is detected.
- Our **objective** is to detect the network factors influencing the failures in the VAT declaration and explain how the behavior propagates.



Scheme of the transaction network

Empirical facts

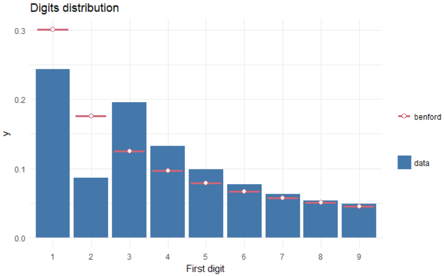
- We use information of VAT declarations in a region of Spain (Canary Islands) during year 2002.

Network	N	$<k>$	γ_{in}	γ_{out}	$\bar{l}$	$\bar{l}_r$
Joint	208 228	3.99	-	-	4.86	8.97
Matched	25 614	3.79	3.40	3.90	6.10	7.72
Differed amount	15 494	2.07	2.48	2.42	6.61	12.44
Non reciprocal buyer	161 934	2.10	2.42	2.56	6.85	15.37
Non reciprocal seller	87 891	2.30	2.21	2.70	8.10	13.17

- For the directed network (seller to buyer), many of them exhibit a power-law fit (significant γ_{in} and γ_{out}).

Empirical facts

- According to the Benford law, it is expected that the first digit d of real accounting data follows the distribution $log\left(\frac{d+1}{d}\right)$.

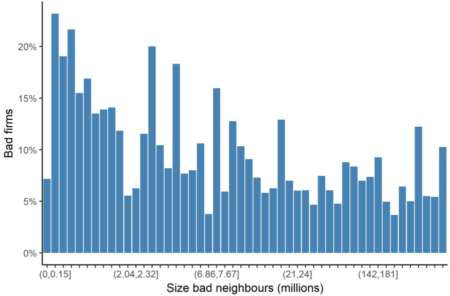


First digit distribution of sales and Benford distribution

- It is observed that data does not follow Benford law.

Empirical facts

- Neighbor's strength may influence on the agent's behavior.



Relationship between the average strength of "bad" neighbors and probability of being bad

- In this case, it is observed a monotonous trajectory.

The model

- We borrow arguments from innovation diffusion models to represent the contagious effect of "bad" behavior among firms in a business network.
- $G = (N, L, \bar{S})$ network, which includes a finite number of agents $N = \{1, 2, ..., n\}$ agents, $L \in N \times N$ interactions and $\bar{S}$ sales (or strengths) in the network.
- $x_i(t) \in \{0, 1\}$, state of the agent i at time t: 0 if the agent declares the true amount (uninfected) and 1 if the agent's declaration is not right (infected).
- The evolution of the percentage of infected nodes $\varphi_k(t)$ follows a common SIS model

$$\dot{\varphi}_k = (1 - \varphi_k)\pi_k^{0-1} - \varphi_k \pi_k^{1-0}. \tag{1}$$

The model

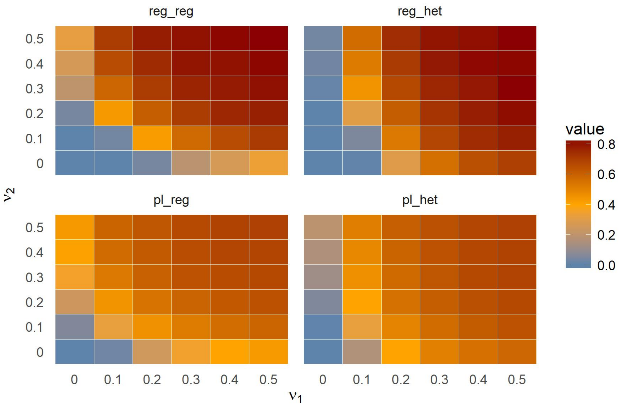
- We assume only two categories of strength $s = \{1, 2\}$. The distribution probability of s is given by $\{q_1, q_2\}$.
- Given a_s the number of infected neighbors with strength s, the rate of diffusion $f(\cdot)$ and recovery $g(\cdot)$ are

$$
\begin{aligned}
&f(a_1, a_2, k) = \nu_1 a_1 + \nu_2 a_2, \\
&g = e, e \in \Re^+
\end{aligned}
$$

- After some calculations, we find that the condition of prevalence of infection is

$$\frac{q_1\nu_1 + q_2\nu_2}{e} \frac{< k^2 >}{< k >} > 1.$$

Simulations



Simulated steady state φ in the innovation diffusion model with node strength: reg-reg: $q_1 = q_2 = \frac{1}{2}$, regular network; pl-reg: $q_1 = q_2 = 0.5$, scale-free network; reg-het: $q_1 = 0.7$, $q_2 = 0.3$, regular network; pl-het: $q_1 = 0.7$, $q_2 = 0.3$, scale-free network. Parameters: $\bar{k} = 3$, $\varphi_0 = 0.1$, $e = 0.2$, $N = 1000$.

Conclusions

- A differentiated structure of matched and unmatched transaction declarations in business networks is observed.
- Data shows positive relationships between neighbor's mean size (strength) and probability to declare badly.
- An innovation diffusion model was proposed to explain the phenomenon, including the influence of agent's strength in the diffusion rate.
- Theoretical and numerical simulations extend the variety of the conditions and level of diffusion.
- Extensions
  - Studying the evolution of the business transaction network.
  - More theoretical insights.